4601. Write sec as $1/\cos$. Then get rid of the inlaid fractions, by multiplying top and bottom of the main fraction by their denominators. Then use a compound-angle formula and also the small-angle approximations for $\sin h$ and $\cos h$.

4602. Find the angles. Then use the cosine rule with the larger angle. You'll need to take the square root of a surd expression. Do this by setting up $(a\sqrt{5}+b)^2 = ...$ and equating coefficients.

4603. Separate the variables, and write the y integrand in partial fractions. Integrate, simplifying the logs. Then exponentiate. Sub the x intercept to find the constant. Make y the subject. At the end, you'll probably need to multiply top and bottom by e^2.

4604. (a) Consider a line of symmetry.
(b) Differentiate and use $y - y_1 = m(x - x_1)$.
(c) Substitute $\left(\frac{3}{4}\pi, 0\right)$ into the equation from (b).
(d) Use N-R or fixed-point iteration. Consider the graph to choose an appropriate starting point.
(e) Having solved for a, the diameter is twice the distance between $(a, \sin a)$ and $\left(\frac{3}{4}\pi, 0\right)$.

4605. A careful explanation serves as a proof here. Start with line two (inclusion), then explain why line three (exclusion) is necessary. Use the concept of overcounting. Then proceed to line four.

4606. Factorising the denominator,

$$\begin{aligned} y &= \frac{1}{(x^2-a^2)^2} \\ &\equiv \frac{1}{(x-a)^2(x+a)^2}. \end{aligned}$$

This is a reciprocal quartic with double asymptotes at $x = \pm a$.

4607. Find the equation, with coefficients in terms of k, of the tangent at $x = 1$. Substitute the point $(3, 0)$ and solve.

4608. Call the radius of the big circle r. Form a right-angled triangle with vertices at the centres of the largest circle, the central unit circle, and one of the medium-sized circles. Set up Pythagoras for this triangle and solve for r.

4609. (a) Multiply the SHM equation by the mass and consider NII.
(b) i. Find the second derivative of the proposed solution curve, and substitute for x.
ii. Use the range to find the amplitude. Then substitute in the conditions $t = 0, x = -2$.

4610. Call the indefinite integral I. Integrate by parts twice, either using the tabular integration method or not. Either way, end up with something of the form

$$I = \mathrm{f}(x) - 4I.$$

Solve for I. Then set up the definite integral.

4611. (a) The equation of the ellipse is

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1.$$

Use the first Pythagorean trig identity.
(b) Find the gradient of the tangent at parameter θ using the parametric differentiation formula. Find the equation of the normal and substitute $y = 0$.

4612. Use a polynomial solver, and the factor theorem.

4613. (a) Set the derivative to zero and then factorise the resulting quadratic in x^3. Evaluate the second derivative at the SPs.
(b) Set $y = 0$ for x intercepts. Factorise and show that there is a sign change at $x = 0$. Explain why, considering the other factors, there can be no other sign change.

4614. (a) The integral gives the area of a quarter-circle.
(b) Use the substitution $x = r\sin\theta$.

4615. Since X has a binomial distribution, it takes only discrete values. So, the required probability is

$$p = \mathbb{P}\left(X \in \{2,3,4\} \mid X \in \{3,4,5\}\right).$$

Use the conditional probability formula.

4616. (a) Substitute the definitions of a and b into $b = a^2$ and simplify.
(b) Substitute $x = \sqrt{2}$, $y = 0$.
(c) Consider the area of the shaded region as the area of the rectangle minus the area between $b = a^2$ and $b = 0$. Think of the line $b = 0$ as the a axis. The (a, b) axes are at 45° to the (x, y) axes.
(d) Just evaluate!

4617. This is a separable differential equation. Both of the resulting integrals are standard results.

4618. Use the parametric integration formula to show that the area is given by

$$\int_0^{2\pi} \tfrac{1}{2}\cos^2 2t + \cos t\cos 2t + \cos^2 t \, dt$$

Integrate the first and third terms in the usual way, using $\cos^2 t \equiv \frac{1}{2}(\cos 2t + 1)$. Integrate the second term with $\cos 2t \equiv 1 - 2\sin^2 t$, then inspection.

4619. Go through the checklist:

① Find any axis intercepts.
② Locate any asymptotes.
③ Find any SPs.
④ Consider the behaviour as $x \to \pm\infty$.
⑤ If needed, classify any SPs.

4620. For each assignation of f, find the probability that $f(x) > 1/2$. This means solving the inequality e.g. $\sin x > 1/2$, and finding the size of the solution set, as compared to $[0, 2\pi)$. Do this for both sin and $\sin^2$. Put this information into a tree diagram to visualise the whole process.

4621. (a) Show that the curve has 1 horizontal and n vertical asymptotes.

(b) Sketch the curve for $n = 3$, noting that each term of the graph is decreasing everywhere, and that each vertical asymptote is a single asymptote. It should become clear where the $n - 1$ roots are.

4622. (a) Edge E_1 is the uppermost. Use basic trig.

(b) Use a parallel line theorem, together with the interior angle of a pentagon.

(c) You already have the position vector of the vertex between E_1 and E_2. Find the direction vector (of unit length) running along E_2 from here. Put the two together with parameter t.

(d) The t domain should be chosen so that E_2 is the same length as E_1.

4623. Use the concave/convex information to establish that the second derivative is zero at $x = 0, 4$. Use the factor theorem to write the second derivative in general algebraic form, integrate this twice, and equate coefficients of x^4 to find the exact algebraic form of the second derivative. Integrate this once and use the fact that there is an SP at $x = 3$ to find the constant of integration. Integrate again, using $(3, -11)$ to find the new constant.

4624. (a) Use the substitution. The lower limit cannot be calculated directly. However, it can be set up using $x = \delta$, where $\delta \to 0$. Let $k = \ln \delta$.

(b) Integrate by parts twice. This is easiest by the tabular integration method.

4625. The idea here is that a quadratic approximation should match the zeroth (the original function), first and second derivatives, a cubic approximation should match the zeroth, first, second and third derivatives, and so on. So, in (b), set up

$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2 + k\theta^3.$$

Do likewise for (c), using the answer to (b).

4626. Sketch the graphs. You should find that there is a line segment in common to both. Express this as a set in the following manner:

$$\{(x, y) \in \mathbb{R}^2 : \text{conditions on } x \text{ and } y\}.$$

Now, you could write $|y| = x+2$ and $|x| = y+2$ as conditions. However, the task of the question, as with most solutions in mathematics, is to simplify as far as possible.

4627. Expand $\left(1 - \frac{1}{2}x^2\right)^{-1}$ binomially.

4628. Differentiate the first equation with respect to x. Substitute and solve for y, showing that it must be constant. Hence, show that x is constant, and explain what problem this causes for the model.

4629. Find the upper y intercept by setting $x = 0$. Set up a definite integral with respect to y for the area in the positive quadrant. Take out a factor of y and integrate by substitution.

4630. Start by considering the three possibilities $k < 0$, $k = 0$ or $k > 0$. Only one is viable.

4631. (a) Differentiate and find $\frac{dy}{dx}$. Substitute the value $x = a - 1$ into this to find the gradient at A. Then use $y - y_1 = m(x - x_1)$.

(b) Substitute $x = y = 0$ into the result of (a).

4632. (a) Consider the length of the sub-interval.

(b) Let p be the probability that a random number lies in $[0.231, 0.629]$. The hypotheses are

$$H_0 : p = 0.398,$$
$$H_1 : p \neq 0.398.$$

It is a two-tailed test, so compare cumulative probabilities from $B(100, 0.398)$ to 0.5%.

4633. The equation of the curve is a quadratic in y. Use the quadratic formula to make y the subject, then set up a definite integral. Get your calculator to evaluate it.

4634. Factorise and solve each equation on its own. Then find all (x, y) points which lie in both solution sets.

4635. (a) The mass of chain on each slope is proportional to the length on that slope. Without loss of generality, let the constant of proportionality be 1, so the total mass is 2. Nudge the chain rightwards, calling the mass on the right-hand slope $1 + x$. The mass on the left-hand slope is $1 - x$. Resolve along the chain, letting the internal tensions cancel.

(b) At first, the acceleration grows exponentially. It is then constant for $x \geq 1$. It is continuous at the change in behaviour.

(c) Find the derivatives of the proposed solution. Verify that it gives the correct initial speed. Then choose k such that it satisfies the DE.

4636. Differentiate $A = \pi r^2$ with respect to t.

4637. Let the position vectors of vertices A, B, C be $\mathbf{a}, \mathbf{b}, \mathbf{c}$, relative to some origin. Find the position vector of a point P which lies $\frac{2}{3}$ of the way along one of the medians, closer to the midpoint than the vertex. Show that this expression is symmetrical in $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and use that symmetry to prove that P lies on all three medians.

4638. Quote the result $\displaystyle\int a^x\, dx = \frac{1}{\ln a} a^x + c$.

4639. Throughout, it is easier to work in column vectors.

(a) Take out common factors of r and then ω when you can.

(b) Take the magnitude of your equation for $\mathbf{v}$.

(c) Differentiate again.

(d) Write $\mathbf{a}$ in terms of $\mathbf{r}$ and ω. Then take the magnitude of the equation and sub $v = r\omega$.

4640. Use two of the double-angle formulae to rewrite the integrand in terms of $\cos 4x$.

4641. One is true, two are false.

4642. The equation of each circle takes the form

$$(x-a)^2 + (y-r)^2 = r^2.$$

Substitute the given points into this equation and solve simultaneously for r.

4643. Write $\cos 4\theta$ as a polynomial in $\cos\theta$. To do this, starting by expressing

$$\cos 4\theta \equiv \cos(2 \cdot 2\theta).$$

Use a double-angle formula (twice). Then sub into the RHS of the original identity and simplify.

4644. (a) The key fact in both is no sign change, due to a squared factor.

(b) Express $f(x)$ as 5 plus a proper fraction.

(c) The direction from which $y = 5$ is approached (that's what is meant by $5^{\pm}$) is a useful tool for sketching here. Draw $y = 5$ and $x = \frac{9}{2}$ first, and fit the curve around them.

4645. Separate the variables by dividing through by $y^2 - y$ and multiplying by $\cos x$. Then write the y integrand in partial fractions and integrate.

Simplify and exponentiate, converting the additive constant into a multiplicative constant. Rearrange to make y the subject.

4646. Differentiate implicitly, and substitute ${}^{dy}/_{dx} = {}^{3}/_{4}$. Make x the subject, and substitute back into the equation of the curve. Solve for y, choosing the point with integer coordinates.

4647. Set up an equation for the intersection. Use the Newton-Raphson method to find the x coordinate to at least 6 significant figures. Then set up two definite integrals either side of this value. Evaluate these with a calculator, or you can integrate them manually.

4648. None of the implication symbols are correct. Look for two counterexamples, one in each direction.

4649. Use a conditioning approach. Choose a marble wlog. Then multiply two probabilities: that the second differs from the first, and that the third differs from the first two.

4650. Rewrite the logarithm as

$$y = \log_k x \equiv \frac{\ln x}{\ln k}.$$

Differentiate this and set the gradient to 1. Then require that the x coordinate generated (in terms of k) give a point on the line $y = x$.

4651. Use the substitution. This should eliminate all mention of the constant a. You can then write the x integrand in partial fractions. Simplify with log rules at the end.

4652. The symmetry of the curves dictates that all points of intersection lie on the lines $y = \pm x$.

4653. (a) Sub $k = 1$.

(b) Simplify each side separately.

(c) What is added to the sum of the first k squared integers to give the sum of the first $k + 1$ squared integers?

4654. Expand each of the factors on the RHS using compound-angle formulae. Multiply out to get four terms. Simplify with

$$\sin 2\theta \equiv 2 \sin\theta \cos\theta.$$

Then use the first Pythagorean trig identity.

4655. To maximise the number of pieces, every new cut must cross every existing cut. Show that the first differences increase linearly, and therefore that the sequence must be quadratic. Find its coefficients in the usual way.

4656. The problem is symmetrical in the line $x = 1$. So, consider a generic tangent to $y = x^2$, at $x = a$. Find two values of a by substituting $(1, -5/4)$ into this. Choose the right one by sketching. Then find the gradient of the relevant tangent and use the symmetry of the problem.

4657. Express the summand in partial fractions. Then write it longhand and cancel terms.

4658. Let $\arctan x = y$, so $\tan y = x$.

4659. Find the equation of a generic normal at $x = \theta$. Approximate the coefficients of this equation using small-angle approximations for both sin and cos. Lastly, set $y = 0$ and solve for x. You should get an expression cubic in θ, which is then negligible for small θ.

4660. Consider the image of

$$y = x^p + x^{p-1}$$

under stretches by scale factor c and d in the x and y directions. Algebraically, this is

$$\begin{aligned} y &= d\left(\frac{x}{c}\right)^p + d\left(\frac{x}{c}\right)^{p-1} \\ &\equiv \frac{d}{c^p}x^p + \frac{d}{c^{p-1}}x^{p-1}. \end{aligned}$$

Equate coefficients. You can find c in terms of a, b by dividing the equations. Substitute back in to find d in terms of a, b and p.

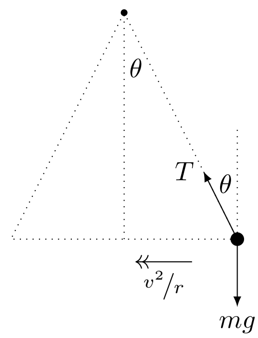
4661. (a) Split the sum into the first term and the rest. Then use the fact that $n(n-1) < n^2$.
(b) Write the summand in partial fractions.

4662. Find the x intercepts. Then, to integrate, call the indefinite integral I. Integrate by parts twice, and rearrange to make I the subject. Hence, show that the areas of the shaded regions are in geometric progression, with common ratio $e^{-\pi}$. Find S_∞ and simplify.

4663. Set the discriminants to zero and solve.

4664. Let the first two terms be a and b. Work out the subsequent terms, and show that $u_7 = u_1$ and $u_8 = u_2$. This shows that the sequence is periodic, with largest possible period 6. The other possible periods are $1, 2, 3$. Show that each of these requires $a = b = 0$.

4665. In cross-section, the force diagram is



Resolve horizontally and vertically. Then find the radius of the circular path in terms of l and θ. Hence, derive an expression for the speed v and the circumference of the circle. Divide these to get the time period.

4666. Use the factorial definition of ${}^n\mathrm{C}_r$. Then, without multiplying out, divide through by the common factors. Justify the fact that they are non-zero.

4667. (a) Sub in and simplify.
(b) Set up the equation. You can't solve directly. But you know that the equation has exactly one root $x \in (-\pi, \pi)$, so the root must be a stationary point.
(c) What does this say about order of composition for f and g?

4668. Assume that $\sqrt{p} + \sqrt{q}$ is rational, and is therefore equal to a/b, where $a, b \in \mathbb{Z}$. Square both sides and rearrange to $\sqrt{pq} = ...$ Seek out a contradiction.

4669. Find the area of a rectangle, and then subtract the area underneath the curve. The integral is a standard one. There's a little bit of log rule manipulation to be done.

4670. Complete the following:
① Setting $y = 0$, the only axis intercept is...
② As $x \to \infty$, $y \to$ ...
③ As $x \to -\infty$, $y \to$ ....
④ Setting the derivative to zero for SPs...

4671. Assume that "consecutive" is not cyclic, i.e. that A does not follow Z.

There are ${}^{26}\mathrm{C}_3$ ways of selecting three letters from the alphabet. Work out how many successful sets there are.

4672. Write out the first few terms, until you see the geometric series which appears. Use the standard partial sum formula.

4673. This is about angle-chasing. There are three main angles involved, all of which are acute:

- α, between reflector and x axis,
- β, between incoming light and reflector,
- γ, between outgoing light and reflector.

Work them out in terms of k, in order α, β, γ.

4674. Since the line $y = kx$ is tangent to the curve at two distinct points, the equation $x^4 - 2x^2 + 2x + 1 = kx$ must have two double roots:

$$x^4 - 2x^2 + (2-k)x + 1 \equiv (x-a)^2(x-b)^2.$$

Equate coefficients.

4675. Using the fact that the derivatives are reciprocals of one another, this is a quadratic in $\frac{dy}{dx}$.

4676. Find the equation of a general normal at (a, a^2). Set an equation for intersections of this normal with the curve itself. Sub $x = 5/\sqrt{8}$ and $y = 25/8$. Solve for a. Not all solutions to this equation will be solutions to the original problem.

4677. The equations are a pair of straight lines in the (x, y) plane. These do not have a unique (x, y) solution point if they are parallel. So, equate the gradients. Simplify and you'll get a quadratic in p and q. The symmetry of the problem dictates that $p = q$ is a root. Take out the relevant factor. Then rearrange what remains to the form $q = \text{f}(p)$, mimicking $y = \text{f}(x)$. Sketch this along with the line $p = q$.

4678. Write the integrand as a constant plus a proper algebraic fraction. Then write in partial fractions, using the form

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}.$$

Integrate term by term and simplify.

4679. Solve the boundary equation using the third Pythagorean trig identity. Then read the solution set from the graph, excluding the x values at the vertical asymptotes. The solution set consists of four distinct intervals.

4680. Use the compound-angle formula

$$\sin(x+h) - \sin(x-h) \equiv 2\cos x \sin h$$

to simplify the numerator. Then take a factor of $\cos x$ out of the limit (it doesn't depend on h). What remains is then a standard result:

$$\lim_{h \to 0} \frac{\sin h}{h} = 1.$$

You can quote this: it is equivalent to the small-angle approximation $\sin\theta \approx \theta$ for small θ.

4681. Find the derivative $\text{f}'(x)$ and substitute into the formula. The integrand simplifies, so you can then integrate directly.

4682. Let the sphere have radius 1. Take a cross-section, placing the apex of the cone/triangle at the point $(0, 1)$. Express the height of the cone in terms of the radius r of its base. Hence, express the volume in terms of r, and optimise using calculus.

4683. Integrate by inspection, noting that $\frac{1}{x+1}$ is the derivative of $\ln(x+1)$.

4684. Find the equation of a general normal at (p, p^2). Call the centre of the circle $(1, k)$. Sub this point into the equation of the normal, and solve for k in terms of p. Then set the squared distance between $(1, k)$ and (p, p^2) to 1. Simplify to form a quartic in p and solve numerically.

4685. Take out a factor of $(x - y)$ from the first equation. Then substitute $x - y = 1$ and $y = x - 1$. Solve the resulting quadratic in x.

4686. Start by writing the integrand as

$$\sin^4 x = \sin^2 x(1 - \cos^2 x).$$

Then use double-angle formulae.

4687. Consider the points at which the mod functions toggle between active and passive.

4688. (a) Explain why $\delta A = h\delta w + w\delta h + \delta w \delta h$.

(b) Consider the fact that δw, δh and δt all tend to zero linearly.

(c) Take the limit as $\delta t \to 0$, turning the average δ rates of change into instantaneous d rates of change.

4689. Use the substitution $x = a\sin\theta$.

4690. The LHS is a product. It is non-negative iff both of its factors are non-negative or both are negative. Sketch the boundary equations $y = x^2$ and $x = y^2$, then work out which regions satisfy the inequality.

4691. The general equation of the trajectory of a particle launched from the origin is

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

4692. Sketch a graph of $y = \arctan x$. Set the range of $\cos x$ as the domain for this graph.

4693. Rewrite as

$$\frac{x^2+1}{x+1} \equiv \frac{x(x+1)-x}{x+1} \equiv x - \frac{x}{x+1}.$$

4694. Take out a factor of $(x-a)$ in both numerator and denominator.

4695. Simplify $\cos 2x \tan 2x$ before you begin.

4696. Find the first derivative in the usual way, using the chain rule. To find the second derivative, use the chain rule again, in this following fashion:

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx}\left(\frac{dy}{dx}\right)\\ &= \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}.\end{aligned}$$

4697. Rewrite the middle inequality as

$$\mathbb{P}(X' \cup Y) > \tfrac{3}{4} \iff \mathbb{P}(X \cap Y') < \tfrac{1}{4}.$$

Then use the conditional probability formula.

4698. Let $p = x + y$ and $q = x - y$.

4699. Use a double-angle identity to simplify the graph. Then consider the non-zero x intercepts, in light of the squared factor that appears.

4700. For parts (a) and (b), the standard technique would involve finding the value of the parameter. But there is a neater way in this case. Find the Cartesian equation by squaring the equations and adding them. The results follow geometrically.

——— End of 47th Hundred ———